MTH 132 Section 103 Exam 3

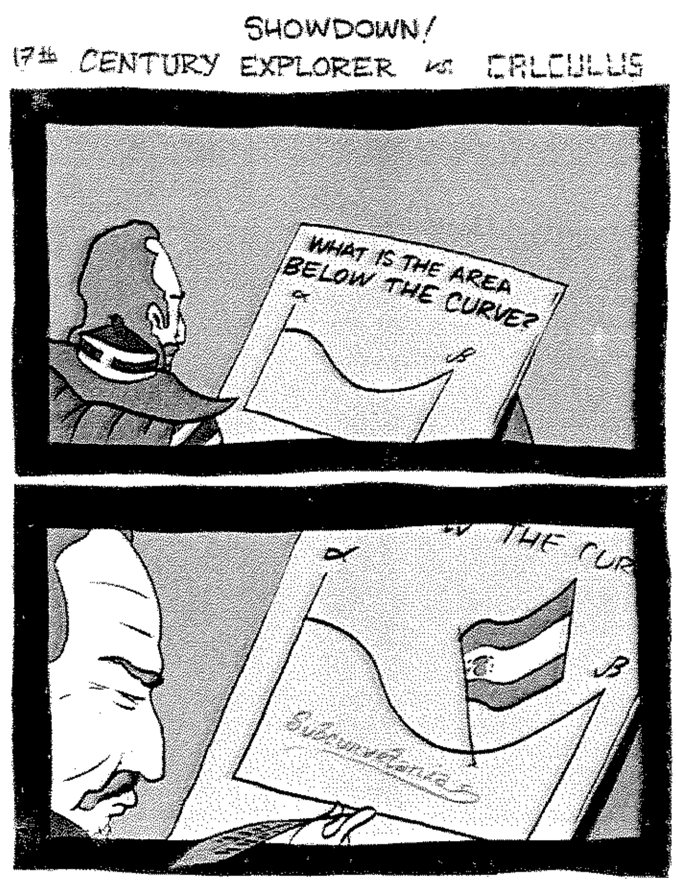
Name: Solutions

Date: 18 June 2014

Do not start this exam until instructed; you will have 50 minutes to finish the exam. Show all your work to receive credit. No notes, books, calculators, phones or electronic devices are allowed on this exam.

There are 7 problems on this exam on 5 pages, in addition to this cover page.

Good luck!



From SMBC.

1. (25 points) Consider the function

$$f(x) = (x-1)(x+2)^2$$

and answer the following questions.

(a) Find the derivatives f' and f''. You may find it helpful to factor the expression for f'.

$$f'(x) = 1(x+2)^2 + 2\cdot(x-1)(x+2) = (x+2)(x+2+2x-2)$$
$$= \boxed{3x(x+2)}$$

$$f''(x) = 3(x+2) + 3x = \boxed{6x+6}$$

(b) Find the critical points of f.

$$f'(x) = 0 \longrightarrow \boxed{x = 0, -2}$$

(c) On what interval(s) is the function increasing? Decreasing?

(-)(-) (-)(+) (+)(+)

-2 0

(+) (-) (+)

Increasing: $(-\infty, -2) \cup (0, \infty)$

Decreasing: $(-2, 0)$

(d) For each critical point, say whether it is a local maximum, minimum, or neither.

By 1st derivative test:

-2 Max

0 Min

(e) Determine where the function is concave up or concave down.

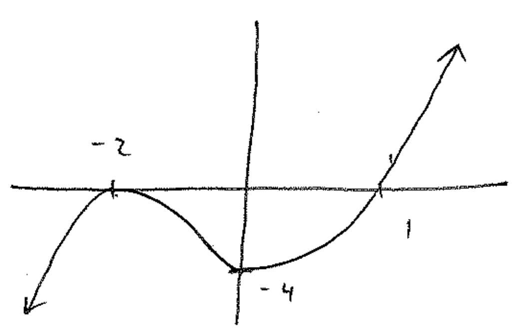
$$f''(x) = 6(x+1)$$

− −1 +

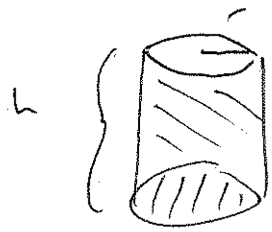
Concave up: $(-1, \infty)$

Concave down: $(-\infty, -1)$

(f) Use the previous parts to give a rough sketch of the graph of f.



2. (24 points) A can is to be constructed in the shape of a right circular cylinder, with an open top. Find the radius that minimizes the surface area of the can, assuming the can has volume 1. Use a test to show that your answer is a minimum.



$$V = \pi r^2 h = 1$$

$$\longrightarrow h = \frac{1}{\pi r^2}$$

$$\text{Area} = \underbrace{\pi r^2}_{\text{bottom}} + \underbrace{2\pi r h}_{\text{side}} = \pi r^2 + \frac{2}{r}$$

$$\boxed{\text{Goal: Maximize } A(r) = \pi r^2 + \frac{2}{r} \text{ for } r > 0}$$

Critical point: $A'(r) = 2\pi r - \frac{2}{r^2} = 0$

$$\pi r^3 - 1 = 0$$

$$\boxed{r = \frac{1}{\sqrt[3]{\pi}}}$$

Check: $A''(r) = 2\pi + \frac{4}{r^3}$

$$A''\left(\frac{1}{\sqrt[3]{\pi}}\right) > 0$$

So it's a minimum by the 2nd Deriv. Test.

3. (24 points) Evaluate the following indefinite integrals.

(a)

$$\int x^3 + x^{10} + \frac{1}{x^4} dx \qquad = x^{-4}$$

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

$$= \boxed{\frac{x^4}{4} + \frac{x^{11}}{11} + \frac{x^{-3}}{-3} + C}$$

(b)

$$\int 2\sin(x) + 12\cos(x) - \frac{1}{\sqrt{x}} dx$$

$$x^{1/2} / {}^{1}/_{2}$$

$$= \boxed{-2\cos x + 12 \sin x - 2x^{1/2} + C}$$

(c)

$$\int \cos(\theta)(\tan(\theta) + \sec(\theta))d\theta$$

$$= \int \cancel{\cos\theta} \left(\frac{\sin\theta}{\cancel{\cos\theta}} + \frac{1}{\cancel{\cos\theta}} \right) d\theta$$

$$= \int \sin\theta + 1 \quad d\theta = \boxed{-\cos\theta + \theta + C}$$

(d)

$$\int (3x+1)^{22} dx$$

Guess $(3x+1)^{23}$

$$\frac{d}{dx}(3x+1)^{23} = 23(3x+1)^{22} \cdot 3$$

divide by $23 \cdot 3$.

$$= \boxed{\frac{(3x+1)^{23}}{3 . 23} + C}$$

Choose any 3 of the following 4 questions to do (9 points each). If you do all 4, the best 3 will be graded.

4. Verify that the conclusion of the mean value theorem holds for the function $f(x) = x^3$ on the interval $[1, 2]$.

$$\frac{f(2) - f(1)}{2 - 1} = \frac{8 - 1}{1} = 7$$

$$f'(x) = 3x^2 = 7$$

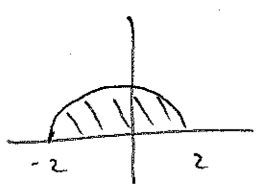
$$x^2 = \frac{7}{3}$$

$$x = \sqrt{\frac{7}{3}}$$

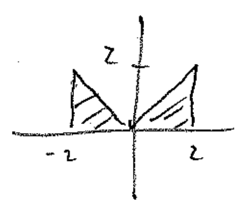
Then $f'\left(\sqrt{\frac{7}{3}}\right) = \frac{f(2) - f(1)}{2 - 1}$.

5. By interpreting definite integrals as areas, evaluate

$$\int_{-2}^{2} \sqrt{4 - x^2} + |x| dx$$



$+$



Semicircle Two triangles

$$= \frac{1}{2}\pi \cdot 2^2 + 2\left(\frac{1}{2} \cdot 2 \cdot 2\right)$$

$$= \boxed{2\pi + 4}$$

6. Use sigma notation to answer the following:

(a) Evaluate $\sum_{k=3}^{5} k-6$ $= (3-6)+(4-6)+(\cancel{7}5-\cancel{8}6)$

$= -3-2-1 = \boxed{-6}$

(b) Write $1+2+4+8+16+32+\cdots+2^{1000}$ in sigma notation.

$2^0 + 2^1 + 2^2 + 2^3 + \cdots + 2^{1000}$

$= \boxed{\sum_{k=0}^{1000} 2^k}$

7. Suppose that f and g are differentiable functions such that $f' = g$ and $g' = -f$; define

$$h(x) = f(x)^2 + g(x)^2$$

If $h(0) = 0$, find $h(1)$. [Hint: What is h'?]

$h'(x) = 2ff' + 2gg'$

$= 2fg + 2g(-f) = 0$

$\longrightarrow$ h constant

$\longrightarrow$ $\boxed{h(1) = h(0) = 0}$